Math 1411 Quiz 8 Name KEY

Extra Credit – due Wednesday, 11/6

There are three problems worth 10 points each. **You must show all of your work to receive full/partial credit.**

1) Find the indefinite integral.

$$\int \frac{x^2 + 2x - 3}{x^4} dx$$

Rewrite: $\int \frac{x^2}{x^4} + \frac{2x}{x^4} - \frac{3}{x^4} dx$

$= \int x^{-2} + 2x^{-3} - 3x^{-4} dx$

$= -x^{-1} - x^{-2} + x^{-3} + C$

$= -\frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^3} + C$

2) Find the indefinite integral.

$$\int \frac{\sin x}{1 - \sin^2 x} dx = \int \frac{\sin x}{\cos^2 x} dx$$

$= \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} dx = \int \tan x \sec x \, dx$

$= \sec x + C$

3) Use the limit process to find the area of the region bounded by the graph of the function and the x-axis over the given interval.

$$y = 2x - x^3, \quad [0, 1]$$

$\Delta x = \frac{1}{n}$ $\quad x_1 = \frac{1}{n}, \; x_2 = \frac{2}{n}, \; \dots, \; x_i = \frac{i}{n}$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{i}{n}\right)\left(\frac{1}{n}\right)$$

$$= \lim_{n\to\infty} \left(\frac{1}{n}\right) \sum_{i=1}^{n} \left[2\left(\frac{i}{n}\right) - \left(\frac{i}{n}\right)^3\right] = \lim_{n\to\infty} \left(\frac{1}{n}\right)\left[\frac{2}{n}\sum_{i=1}^{n} i - \frac{1}{n^3}\sum_{i=1}^{n} i^3\right]$$

$$= \lim_{n\to\infty} \left(\frac{1}{n}\right)\left[\frac{\cancel{2}}{\cancel{n}}\left(\frac{\cancel{n}(n-1)}{\cancel{2}}\right) - \frac{1}{n^{\cancel{3}}}\left(\frac{\cancel{n^2}(n+1)^2}{4}\right)\right]$$

$$= \lim_{n\to\infty} \left[\frac{n-1}{n} - \frac{(n+1)^2}{4n^2}\right] = \lim_{n\to\infty} \left[\frac{n-1}{n} - \frac{n^2+2n+1}{4n^2}\right]$$

$$= 1 - \frac{1}{4} = \frac{3}{4}$$